

COMPUTER EVALUATION OF DOUBLE-THEODOLITE DATA

by

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ABSTRACT

Conventional methods of evaluating double-theodolite data involve numerical and/or graphical techniques that are laborious and time consuming. The following presentation offers a method whereby the electronic digital computer first computes the position of the balloon in space and then from this determines the wind vector. The program also makes provision for the problem of missing data.

This computing technique is highly valuable when a large number of pilot balloon ascents must be processed.

While conducting field operations at the Enrico Fermi Nuclear Reactor Site during the summer of 1961, it was necessary to take double-theodolite balloon ascents as a means of evaluating the wind speed in the lower few thousand feet of the atmosphere. The basic approaches to evaluating double-theodolite data have been: 1) numerically to calculate the position of the balloon in space and time, and then to plot it on a winds aloft plotting board to determine wind speed and direction; or 2) to use graphical methods throughout to determine wind speed and direction. Either of these methods requires the use of winds aloft plotting boards and both methods are laborious and time consuming. Weedfall and Jagodzinski [1961] describe a graphical method in which they can evaluate a 30-interval run in about 25 min. The United States Weather Bureau Circular "O" describes a method which takes 70 min for a 30-interval run. When a large number of observations are to be evaluated, the high speed electronic digital computer offers a much more convenient and rapid technique of analysis.

The following method of analysis requires no plotting of data. The only work is in the conversion of the data from

tabular form to punched cards or any other convenient form of computer input. The data deck consists of one card containing the length of the baseline and the number of observations, followed by four sets of cards. The first of these contains the azimuth angles from station A (the balloon release point), the second contains the elevation angles from station A, the third contains the azimuth angles from station B (the satellite station), and the fourth contains the elevation angles from station B. These angles are read into a three dimensional matrix A_{ijk} , where in this case i is the number of layers, j is the number of rows, and k is the number of columns.

The output of the computer is in the desired form of the horizontal wind vector and the altitude of the balloon for each sounding. Also printed out are the horizontal radial distance of the balloon from the launch site, the u and v components of the wind vector, and the x and y coordinates of the balloon. The mathematical basis for the method presented is simple and is easily handled by the digital computer.

The use of two theodolites necessitates a baseline which is carefully surveyed. The baseline used was 2000 ft long and lay in an east-west direction. Since the terrain

is relatively flat, the two stations were at equal elevations, except for the two kilometer inland site which had seven feet difference in elevation. The other sites had less than a foot difference in elevation. Communications were maintained through the use of two transistorized walkie-talkies. Five men were used, three at the balloon release station and two at the satellite station (figure 1). At the balloon release station one man tracked the balloon and read the two angles, one man recorded, and one man kept time. At the satellite station the radio was turned up so both could hear. One man tracked the balloon and read the angles while the second man recorded the data. This arrangement allowed for reading the theodolites at ten second intervals.

Three sites were surveyed with 2000 ft baselines, one at the plant site, one at two kilometers inland, and one at four kilometers inland. Provisions were also made to survey an inland site at six kilometers which will be done during the spring of 1962.

The two theodolites were adjusted so that both were reading north at 360° . As long as the balloon is not too near the vertical plane containing the baseline, the horizontal triangle ABC (figure 2) is solved first, to obtain the projection of the balloon and its horizontal distance



Figure 1. Double-theodolite tracking operations at the
satellite station.

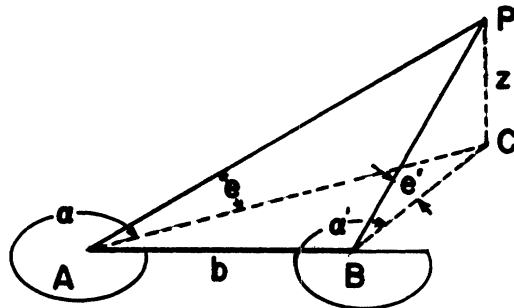


Figure 2. Perspective of two theodolite triangulation.

from the two stations. AB is the baseline of length b ;
 P is the position of the balloon, and C is its projection
on the horizontal plane through A . The angles measured
from A are α and ϵ , those measured from B , α' and
 ϵ' . After the projection of the balloon (C) has been
obtained, its height above the level of A is calculated by
the relation $Z = AC \tan \epsilon$, where ϵ is the elevation angle
measured at A . The elevation above B may be calculated
as a check; it is given by $Z' = BC \tan \epsilon'$.

It is seen from figure 2

$$\text{angle } ACB = \alpha - \alpha'$$

and by the sine law

$$\frac{AC}{\sin \alpha'} = \frac{b}{\sin (\alpha - \alpha')}$$

therefore

$$AC = \frac{b \sin \alpha'}{\sin (\alpha - \alpha')}$$

Similarly

$$BC = \frac{b \sin \alpha}{\sin (\alpha - \alpha')}$$

also

$$Z = \frac{b \sin \alpha' \tan \epsilon}{\sin (\alpha - \alpha')}$$

and

$$Z' = \frac{b \sin \alpha \tan \epsilon'}{\sin (\alpha - \alpha')}$$

Since in all but one case, A and B are at the same elevation, Z and Z' should be equal. At the two kilometer site where A and B are not in the same horizontal plane, all the above formulae hold except for Z' where it will differ by h, where h is the difference in elevation of A and B.

The analysis so far has described the position of the balloon in space in a cylindrical coordinate system (r, θ , z), where r = AC, $\theta = \alpha$, and z = Z. For the

following approach this needs to be converted to a Cartesian coordinate system (x, y, z) . This is done as follows:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$

The Cartesian coordinate system is set up by requiring that the x-axis coincide with the baseline used for the two theodolites and that the origin be at the theodolite where the balloon is released. The baseline was located on an east-west line thus making the y-axis on a north-south line (figure 3) .

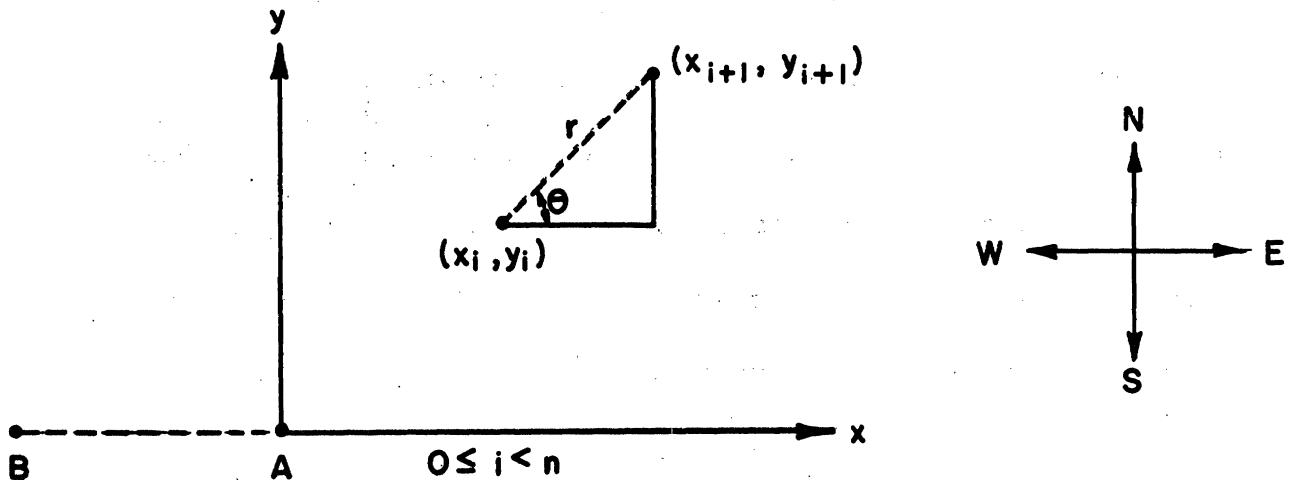


Figure 3. Diagram showing coordinate system used in equations.

The position of the balloon in the x , y plane is designated as x_0 , y_0 for the release position; x_1 , y_1 as the first position, etc., to x_n , y_n as the n^{th} position; then in order to find the distance the balloon traveled in going from x_i , y_i to x_{i+1} , y_{i+1} the following formulae are used

$$r = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$$

$$\theta = \tan^{-1} \left[\frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right]$$

$$0 \leq i < n$$

Thus, knowing r , the distance traveled, and t , the time between successive readings (for this program it was every 10 sec), the average wind speed \bar{U} is simply r/t . The height of the balloon was previously calculated by the computer in order to describe its position in space. The U. S. Weather Bureau in computing its pibal soundings uses every other point in obtaining a wind speed and direction; this is very simply done by using x_{i+2} , y_{i+2} instead of x_{i+1} , y_{i+1} .

When the computer comes to a missing point, an average is taken by using the next point. This is also true if two consecutive points are missing. However, if three or more consecutive points are missing, then an average is not taken and the computer continues to test for missing data until points are again found and the averaging process starts again. Thus a segment of the run is considered missing only if three or more consecutive readings are missing.

A problem also arises when the balloon crosses the baseline or travels along it. A small error in α or α' will cause a large error in the computed position of the balloon. In this case it is better to solve the vertical triangles ABP and ACP (figure 4). It follows that

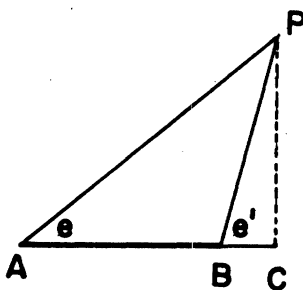


Figure 4. Solution of vertical triangles.

$$AC = b \frac{\sin \epsilon' \cos \epsilon}{\sin (\epsilon \pm \epsilon')}$$

$$BC = b \frac{\cos \epsilon' \sin \epsilon}{\sin (\epsilon \pm \epsilon')}$$

$$Z = b \frac{\sin \epsilon' \sin \epsilon}{\sin (\epsilon \pm \epsilon')}$$

The positive sign will be used when C lies in AB, the negative sign when it lies in AB produced.

Once the position of the balloon in space is computed, the coordinates are converted to Cartesian and the wind speed and direction are computed as before.

Since the angles as recorded by the theodolites are used only in sine and cosine functions, the fact that they go through 360° makes no difference. The sine and cosine functions are symmetric and periodic, thus the $\sin 362^\circ = \sin 2^\circ$, and the same is true for the cosine.

Since the arc tangent is multivalued, there is a question of which value to choose. This problem can be easily solved by looking at the combination of signs on the x and y values used to compute the arc tangent. If both are positive then the angle lies in the first quadrant; if y is positive and x is negative then the angle lies in the second quadrant; etc. This was even less of a problem for this program since

the computer has a calling subroutine that will compute the arc tangent for values between $0 \rightarrow 2 \pi$.

Just before the results are printed out by the computer, the complement of each angle is taken and then rotated 270° . The complement is taken since the program computes θ in a counterclockwise direction, whereas it is standard practice to use θ in a clockwise direction to express wind direction. The angle is then rotated 270° because in the program 360° is east, and the angle θ defines the direction in which the balloon is traveling. Thus θ is rotated 90° to correspond to the concept of north being 360° and then 180° more to correspond to the concept of wind direction being given as the direction from whence the wind is blowing.

Table 1 shows a comparison between the computer evaluation and the hand calculated values. As a means of showing the computer evaluation with missing data, two values were read in as zeros and the results compared. The first pair of columns presents the radial horizontal distance from the release point to the balloon (in feet). The second pair gives the height of the balloon above the theodolite (in feet) and the third gives the local wind speed (in feet per second). The fourth pair of columns shows the local wind direction (in

TABLE 1

Comparison of values obtained by the
computer with those obtained by conventional methods.

R		Z		\bar{U}		θ		u	v
(ft)		(ft)		(ft/sec)		(deg)		(ft/sec)	
Comp	HC	Comp	HC	Comp	HC	Comp	HC	Comp	Comp
0	0	0	0						
				7	7	69	69	6.6	2.5
70	70	42.3	42.3						
				5	5	94	92	4.8	-0.3
116	116	64.3	64.3						
				6	6	75	75	6.0	1.6
178	178	96.3	96.3						
0	-	0	-						
				18	18	75	76	17.5	4.5
0	-	0	-						
722	722	329.0	329.0						
				15	14	73	67	14.7	4.4
875	875	382.3	382.3						
				11	10	64	62	10.1	4.9
1118	1118	422.3	422.3						
				13	14	68	67	12.3	4.9
1306	1306	453.8	453.8						
				19	20	67	67	17.4	7.5
1438	1438	509.3	509.3						
				13	12	65	65	12.0	5.6
1588	1588	537.6	537.6						

R - Radial horizontal distance of balloon from launch site (ft)
 Z - Height of balloon (ft)
 \bar{U} - Wind speed (ft/sec)
 θ - Azimuth of wind (deg)
 u - x-comp of wind (as obtained by computer) (ft/sec)
 v - y-comp of wind (as obtained by computer) (ft/sec)
 Comp - Computer values
 HC - Hand calculated values

degrees), and the last two columns give the local u and v components of the wind speed as evaluated by the computer.

The method of evaluation used to obtain the hand calculated figures was as follows: first the position of the balloon in space was numerically calculated, and then this was plotted on a winds aloft plotting board and a graphical analysis used to determine the wind speed and direction.

The program, since it is a computer analysis, suffers from none of the human errors involved in a graphical analysis and is therefore exact. Thus the accuracy of the final result is determined only by the accuracy with which the original angles were recorded.

The program has (as an external function) a system whereby a smoothing subroutine is incorporated in it. Any method of smoothing may be used in the subroutine, with the present smoothing function given by

$$A_i = \frac{1}{4} B_{i-1} + \frac{1}{2} B_i + \frac{1}{4} B_{i+1}$$

where

$$0 < i < n$$

The smoothing subroutine also takes care of missing data.

In the present program only the wind speed is smoothed.

The merit of the program is the great speed with which a large number of data can be analyzed. One program consisted of 10 pibal soundings, each sounding 10 min long with readings taken every 10 sec for a total of 600 points. The computer time involved was 1.5 min for the IBM 709; it required about 45 min of one person's time for punching the data on cards. This is to be compared with time for the method presented by Weedfall and Jagodzinski [1961] which required about 25 min to complete one run of only 30 points and that for the Circular "O" method which requires 70 min for a 30-point run.

The program was written in the MAD* language and is reproduced here in that form in the Appendix. The program can be broken into four main groups--the first group consists of reading in the data and converting to radians. The second group computes the location of the balloon in space. The third group tests for missing data, computes wind speed and direction, and smoothes the wind speed. The last group consists of input and output statements necessary for the MAD translator.

* Michigan Algorithm Decoder

Since the MAD language may not be familiar to everyone, each MAD statement is given with its SAP* translation. Thus the program may be used in most of the standard computers available today.

* Share Assembly Program

REFERENCES

1. Middleton, W. E. K., and A. F. Spilhaus, 1953: Meteorological Instruments (3rd ed.). Toronto, University of Toronto Press, 186-187.
2. Weedfall, R. O., and W. M. Jagodzinski, 1961: Comments on Double-Theodolite Evaluations. Bull. Amer. Meteor. Soc., 42, 322-324.
3. Hansen, F. V , and N. H. Taft, 1959: Another Method of Evaluating Double-Theodolite Runs. Bull. Amer. Meteor. Soc., 40, 221-224.
4. U. S. Weather Bureau Circular "O."

A P P E N D I X

START	PRINT FORMAT JUMP	*001
	READ FORMAT CON,N,B,LIC	*002
	PRINT FORMAT UU	*003
	READ FORMAT DATA,A(1,1,1)...A(1,1,N)	*004
	READ FORMAT DATA,A(1,2,1)...A(1,2,N)	*005
	READ FORMAT DATA,A(2,1,1)...A(2,1,N)	*006
	READ FORMAT DATA,A(2,2,1)...A(2,2,N)	*007
	EXECUTE ZERO.(C(1)...C(N))	*008
	THROUGH S, FOR I=1,1,I.G.N	*009
	WHENEVER A(1,1,I).E.0.OR.A(2,1,I).E.0,TRANSFER TO S	*010
	A(1,1,I)=(90.-A(1,1,I))*6.2831853/360.	*011
	A(1,2,I)=A(1,2,I)*6.2831853/360	*012
	A(2,1,I)=(90.-A(2,1,I))*6.2831853/360.	*013
	A(2,2,I)=A(2,2,I)*6.2831853/360	*014
	C(I)=B/SIN.(A(1,1,I)-A(2,1,I))	*015
S	CONTINUE	*016
	THROUGH SS, FOR I=1,1,I.G.N	*017
	AC(I)=C(I)*SIN.(A(2,1,I))	*018
	RC=C(I)*SIN.(A(1,1,I))	*019
	ZA(I)=AC(I)*SIN.(A(1,2,1))/COS.(A(1,2,1,I))	*020
	ZB(I)=BC*SIN.(A(2,2,1))/COS.(A(2,2,1,I))	*021
	XA(I)=AC(I)*COS.(A(1,1,I))	*022
SS	YA(I)=AC(I)*SIN.(A(1,1,I))	*023
	EXECUTE ZERO.(C(1)...C(N))	*024
	THROUGH MM, FOR J=1,1,J.G.N	*025
	WHENEVER XA(J).E.0.AND.YA(J).E.0	*026
	WHENEVER XA(J+1).E.0.AND.YA(J+1).E.0	*027
	WHENEVER XA(J+2).E.0.AND.YA(J+2).E.0	*028
	THROUGH AA, FOR K=J+3,1,K.G.N	*029
	WHENEVER XA(K).NE.0.OR.YA(K).NE.0	*030
	J=K	*031
	TRANSFER TO MM	*032
AA	END OF CONDITIONAL	*033
	END OF CONDITIONAL	*034
	X=XA(J+2)-XA(J-1)	*035
	Y=YA(J+2)-YA(J-1)	*036
	J=J+2	*037
	TRANSFER TO NN	*038
	END OF CONDITIONAL	*039
	X=XA(J+1)-XA(J-1)	*040
	Y=YA(J+1)-YA(J-1)	*041
	J=J+1	*042
	OTHERWISE	*043
	X=XA(J)-XA(J-1)	*044
	Y=YA(J)-YA(J-1)	*045
NN	END OF CONDITIONAL	*046
	R=SQRT.(X*X+Y*Y)	*047
	THETA=270.-ATN1.(Y,X)*360./6.2831853	*048
	WHENEVER THETA.L.0,THETA=360.+THETA	*049
	PRINT FORMAT PP,J,R,THETA,ZA(J),ZB(J),ZA(J)-ZA(J-1),ZB(J)-ZB(J-1)	*050

1J-1),ZA(J)-ZB(J),X,Y,XA(J),YA(J)	*050
C(IJ)=R	*051
CONTINUE	*052
PRINT FORMAT XX,AC(1)...AC(N)	*053
PUNCH FORMAT DATA1,N,C(1)...C(N)	*054
WHENEVER LIC.NE.O.	*055
EXECUTE SMOTH.(C,ZB,N)	*056
PRINT FORMAT XX,ZB(1)...ZB(N)	*057
END OF CONDITIONAL	*058
TRANSFER TO START	*059
INTEGER N,I,J,K	*060
VECTOR VALUES DIM=3,1,2,200	*061
VECTOR VALUES CON=\$I5,2F10.0*\$	*062
VECTOR VALUES DATA1=\$I5/(16F5.1)*\$	*063
VECTOR VALUES DATA=\$I6F5.1*\$	*064
VECTOR VALUES XX=\$IHO/(1H ,14F8.1,F7.1)*\$	*065
VECTOR VALUES UU=\$IH4,S4,1HJ,S7,1HR,S9,1HO,S8,2HZA,S8,2HZB,S7	*066
1,5HZA-ZA,S5,5HZB-ZB,S5,5HZA-ZB,S7,1HU,S9,1HV,S9,1HX,S9,1HY/1H	*066
2+,S22,1H-*\$	*066
VECTOR VALUES PP=\$IH ,I5,11F10.1*\$	*067
DIMENSION A(1000,DIM),C(200),XA(200),YA(200),ZA(200),ZB(200),	*068
IAC(200)	*068
VECTOR VALUES JUMP=\$IH1*\$	*069
END OF PROGRAM	*070

MAD PROGRAM, TYPE 12 MAR 1962 (ALL NUMBERS ARE OCTAL)

NO. OF LOCATIONS 05601 TRA VECTOR SIZE 00013 TRA VECTOR STARTS 00000 ENTRY PT. 04351 ERASABLE STARTS 77777

VARIABLE STORAGE									
AA	00013	AC	00324	A	02275	BC	02276	B	02277
CON	02301	C	02612	DATA1	02614	DATA	02616	DIM	02622
I	02623	J	02624	JUMP	02625	K	02626	LIC	02627
MM	02630	NN	02631	N	02632	PP	02635	R	02636
S	02637	SS	00013	START	02640	THETA	02641	UU	02664
XA	03175	X	03176	XX	03202	YA	03513	Y	03514
ZA	04025	ZB	04336						

FUNCTION DICTIONARY									
AIN1	00000	COS	00001	MTX	00002	-PRINT	00003	-PUNCH	00004
*READ	00005	SIN	00006	SMOTH	00007	SQRT	00010	SYSTEM	00011
ZERO	00012								

ABSOLUTE CONSTANTS									
04337	+000000000000	04340	+000000000001	04341	+000000000002	04342	+000000000003	04343	+0000000000350
04344	+203622077324	04345	+207550000000	04346	+211416000000	04347	+211550000000	04350	+2330000000000

STATEMENT DICTIONARY									
02630	TXL -305531005530	02631	TXL -305416005404	02637	TXL -304745004744	02640	TXL -304362004354		

PROGRAM									
PRINT FORMAT JUMP									
04351	TSX +0 07400 4 00003	04352	STR -1 00001 0 02625	04353	STR -1 00000 0 00000	*001			
START READ FORMAT CON,N,B,LIC									
04354	TSX +0 07400 4 00005	04355	STR -1 00001 0 02301	04356	STR -1 00000 0 02632	04357	STR -1 00000 0 02277		
04360	STR -1 00000 0 02627	04361	STR -1 00000 0 00000						
PRINT FORMAT UU									
04362	TSX +0 07400 4 00003	04363	STR -1 00001 0 02664	04364	STR -1 00000 0 00000	*003			
READ FORMAT DATA,A(1,1),...A(1,1,N)									
04365	TSX +0 07400 4 00005	04366	STR -1 00001 0 02616	04367	TSX +0 07400 4 00002	04370	TXH +3 02622 0 02275		
04371	TXH +3 00000 0 04340	04372	TXH +3 00000 0 04340	04373	TXH +3 00000 0 02632	04374	SUB +0 40200 0 04375		
04375	TXH +3 00000 0 02275	04376	ALS +0 76700 0 00022	04377	STD +0 62200 0 04410	04400	TSX +0 07400 4 00002		
04401	TXH +3 02622 0 02275	04402	TXH +3 00000 0 04340	04403	TXH +3 00000 0 04340	04404	TXH +3 00000 0 04340		
04405	SUB +0 40200 0 04406	04406	TXH +3 00000 0 02275	04407	STA +0 62100 0 04410	04410	STR -1 00000 0 00000		
04411	STR -1 00000 0 00000								
READ FORMAT DATA,A(1,2,1),...A(1,2,N)									
04412	TSX +0 07400 4 00005	04413	STR -1 00001 0 02616	04414	TSX +0 07400 4 00002	04415	TXH +3 02622 0 02275		
04416	TXH +3 00000 0 04340	04417	TXH +3 00000 0 04341	04420	TXH +3 00000 0 02632	04421	SUB +0 40200 0 04422		
04422	TXH +3 00000 0 02275	04423	ALS +0 76700 0 00022	04424	STD +0 62200 0 04435	04425	TSX +0 07400 4 00002		
04426	TXH +3 02622 0 02275	04427	TXH +3 00000 0 04340	04430	TXH +3 00000 0 04341	04431	TXH +3 00000 0 04340		
04432	SUB +0 40200 0 04433	04433	TXH +3 00000 0 02275	04434	STA +0 62100 0 04435	04435	STR -1 00000 0 00000		
04436	STR -1 00000 0 00000								
READ FORMAT DATA,A(2,1,1),...A(2,1,N)									
04437	TSX +0 07400 4 00005	04440	STR -1 00001 0 02616	04441	TSX +0 07400 4 00002	04442	TXH +3 02622 0 02275		
04443	TXH +3 00000 0 04341	04444	TXH +3 00000 0 04340	04445	TXH +3 00000 0 02632	04446	SUB +0 40200 0 04447		
04447	TXH +3 00000 0 02275	04450	ALS +0 76700 0 00022	04451	STD +0 62200 0 04462	04452	TSX +0 07400 4 00002		
04453	TXH +3 02622 0 02275	04454	TXH +3 00000 0 04341	04455	TXH +3 00000 0 04340	04456	TXH +3 00000 0 04340		
04457	SUB +0 40200 0 04460	04460	TXH +3 00000 0 02275	04461	STA +0 62100 0 04462	04462	STR -1 00000 0 00000		
04463	STR -1 00000 0 00000								
READ FORMAT DATA,A(2,2,1),...A(2,2,N)									
04464	TSX +0 07400 4 00005	04465	STR -1 00001 0 02616	04466	TSX +0 07400 4 00002	04467	TXH +3 02622 0 02275		
04470	TXH +3 00000 0 04341	04471	TXH +3 00000 0 04341	04472	TXH +3 00000 0 02632	04473	SUB +0 40200 0 04474		
04474	TXH +3 00000 0 02275	04475	ALS +0 76700 0 00022	04476	STD +0 62200 0 04507	04477	TSX +0 07400 4 00002		
04500	TXH +3 02622 0 02275	04501	TXH +3 00000 0 04341	04502	TXH +3 00000 0 04341	04503	TXH +3 00000 0 04340		
04504	SUB +0 40200 0 04505	04505	TXH +3 00000 0 02275	04506	STA +0 62100 0 04507	04507	STR -1 00000 0 00000		

04767 TSX +0 07400 4 00006	04770 TXH +3 00000 0 00000	04772 STD +0 60100 0 05577
04773 LDQ +0 56000 1 02612	04774 FMP +0 26000 0 05577	04776 STD +0 60100 1 00324
RC=C(I)*SIN.(A(1,1,1))		
04777 TSX +0 07400 4 00002	05000 TXH +3 02622 0 02275	*019
05003 TXH +3 00000 0 02623	05004 SUB +0 40200 0 05005	05002 TXH +3 00000 0 04340
05007 TSX +0 07400 4 00006	05010 TXH +3 00000 0 00000	05006 STA +0 62100 0 05010
05013 LDQ +0 56000 1 02612	05014 FMP +0 26000 0 05577	05012 STD +0 60100 0 05577
ZAI(1)=AC(I)*SIN.(A(1,2,1))/COS.(A(1,2,1))		
05016 TSX +0 07400 4 00002	05017 TXH +3 02622 0 02275	*020
05022 TXH +3 00000 0 02623	05023 SUB +0 40200 0 05024	05021 TXH +3 00000 0 04341
05026 TSX +0 07400 4 00001	05027 TXH +3 00000 0 00000	05025 STA +0 62100 0 05027
05032 TXH +3 02620 0 02275	05033 TXH +3 00000 0 04340	05031 TSX +0 07400 4 00002
05036 SUB +0 40200 0 05037	05037 TXH +3 00000 0 02275	05035 TXH +3 00000 0 02623
05042 TXH +3 00000 0 00000	05043 LXA +0 53400 1 02623	05041 TSX +0 07400 4 00006
05046 FMP +0 26000 0 05576	05047 FDP +0 24100 0 05577	05045 LDQ +0 56000 1 00324
ZBI(1)=BC*SIN.(A(2,2,1))/COS.(A(2,2,1))		
05052 TSX +0 07400 4 00002	05053 TXH +3 02622 0 02275	*021
05056 TXH +3 00000 0 02623	05057 SUB +0 40200 0 05060	05055 TXH +3 00000 0 04341
05062 TSX +0 07400 4 00001	05063 TXH +3 00000 0 00000	05061 STA +0 62100 0 05063
05066 TXH +3 02622 0 02275	05067 TXH +3 00000 0 04341	05065 TSX +0 07400 4 00002
05072 SUB +0 40200 0 05073	05073 TXH +3 00000 0 02275	05071 TXH +3 00000 0 02623
05076 TXH +3 00000 0 00000	05077 STD +0 60100 0 05576	05075 TSX +0 07400 4 00006
05102 FDP +0 24100 0 05577	05103 LXA +0 53400 1 02623	05101 FMP +0 26000 0 05576
XAI(1)=AC(I)*SIN.(A(1,1,1))		
05105 TSX +0 07400 4 00002	05106 TXH +3 02622 0 02275	*022
05111 TXH +3 00000 0 02623	05112 SUB +0 40200 0 05113	05110 TXH +3 00000 0 04340
05115 TSX +0 07400 4 00001	05116 TXH +3 00000 0 00000	05114 STA +0 62100 0 05116
05121 LDQ +0 56000 1 00324	05122 FMP +0 26000 0 05577	05120 STD +0 60100 0 05577
SS YAI(1)=AC(I)*SIN.(A(1,1,1))		
05125 TSX +0 07400 4 00002	05126 TXH +3 02622 0 02275	*023
05131 TXH +3 00000 0 02623	05132 SUB +0 40200 0 05133	05130 TXH +3 00000 0 04340
05135 TSX +0 07400 4 00006	05136 TXH +3 00000 0 00000	05134 STA +0 62100 0 05136
05141 LDQ +0 56000 1 00324	05142 FMP +0 26000 0 05577	05140 STD +0 60100 0 05577
05145 TRA +0 02000 0 04750		05144 STD +0 60100 1 03513
EXECUTE ZERO.(C(1),...C(N))		
05146 CLA +0 50000 0 02632	05147 SUB +0 40200 0 05150	*024
05152 STD +0 62200 0 05154	05153 TSX +0 07400 4 00012	05151 ALS +0 76700 0 00022
THROUGH MM, FOR J=1,1,J-G-N		
05155 CLA +0 50000 0 04340	05156 STD +0 60100 0 02624	*025
05161 ADD +0 40000 0 04340	05162 STD +0 60100 0 02624	05160 CLA +0 50000 0 02624
05165 TZE +0 10000 0 05167	05166 TPL +0 12000 0 05531	05164 SUB +0 40200 0 02632
WHENEVER XA(J),E.O.AND,YA(J),E.O		
05167 LXA +0 53400 1 02624	05170 CLA +0 50000 0 04337	*026
05173 CHS +0 76000 0 00002	05174 ADD +0 40000 1 03513	05172 FAD +0 30000 0 04350
05177 CLA +0 50000 0 04337	05200 ORA -0 50100 0 04350	05176 LXA +0 53400 1 02624
05203 ADD +0 40000 1 03175	05204 TNZ -0 10000 0 05366	05202 CHS +0 76000 0 00002
WHENEVER XA(J+1),E.O.AND,YA(J+1),E.O		
05205 CLA +0 50000 0 02624	05206 ADD +0 40000 0 04340	*027
05211 ORA -0 50100 0 04350	05212 FAD +0 30000 0 04350	05210 CLA +0 50000 0 04337
05215 TNZ -0 10000 0 05336	05216 CLA +0 50000 0 02624	05214 ADD +0 40000 1 03513
05221 CLA +0 50000 0 04337	05222 ORA -0 50100 0 04350	05220 PAX +0 73400 1 00000
05225 ADD +0 40000 1 03175	05226 TNZ -0 10000 0 05336	05224 CHS +0 76000 0 00002
WHENEVER XA(J+2),E.O.AND,YA(J+2),E.O		
05227 CLA +0 50000 0 02624	05230 ADD +0 40000 0 04341	*028
05233 ORA -0 50100 0 04350	05234 FAD +0 30000 0 04350	05232 CLA +0 50000 0 04337
05237 TNZ -0 10000 0 05306	05240 CLA +0 50000 0 02624	05236 ADD +0 40000 1 03513
05243 CLA +0 50000 0 04337	05244 ORA -0 50100 0 04350	05242 PAX +0 73400 1 00000
05247 ADD +0 40000 1 03175	05250 TNZ -0 10000 0 05306	05246 CHS +0 76000 0 00002
THROUGH AA, FOR K=J+3,1,K-G-N		
05251 CLA +0 50000 0 02624	05252 ADD +0 40000 0 04342	*029
		05254 TRA +0 02000 0 05260

05255 CLA +0 50000 0 02626	05256 ADD +0 40000 0 04340	05257 STD +0 60100 0 02626	05260 CLA +0 50000 0 02626
05261 SUB +0 40200 0 02632	05262 TZE +0 10000 0 05264	05263 TPL +0 12000 0 05306	
WHENEVER XA(K).NE.O.OR.YA(K).NE.O			
05264 LXA +0 53400 1 02626	05265 CLA +0 50000 0 04337	05266 ORA -0 50100 0 04350	05267 FAD +0 30000 0 04350
05270 CHS +0 76000 0 00002	05271 ADD +0 40000 1 03513	05272 TNZ -0 10000 0 05302	05273 LXA +0 53400 1 02626
05274 CLA +0 50000 0 04337	05275 ORA -0 50100 0 04350	05276 FAD +0 30000 0 04350	05277 CHS +0 76000 0 00002
05300 ADD +0 40000 1 03175	05301 TZE +0 10000 0 05305		
J=K			
05302 CLA +0 50000 0 02626	05303 STD +0 60100 0 02624		*031
TRANSFER TO MM			
05304 TRA +0 02000 0 02630			*032
AA			
05305 TRA +0 02000 0 05255			*033
END OF CONDITIONAL			
X=XA(J+2)-XA(J-1)			
05306 CLA +0 50000 0 02624	05307 SUB +0 40200 0 04340	05310 STD +0 60100 0 05577	05311 CLA +0 50000 0 02624
05312 ADD +0 40000 0 04341	05313 PAX +0 73400 1 00000	05314 LXA +0 53400 2 05577	05315 CLA +0 50000 1 03175
05316 FSB +0 30200 2 03175	05317 STD +0 60100 0 03176		
Y=YA(J+2)-YA(J-1)			
05320 CLA +0 50000 0 02624	05321 SUB +0 40200 0 04340	05322 STD +0 60100 0 05577	05323 CLA +0 50000 0 02624
05324 ADD +0 40000 0 04341	05325 PAX +0 73400 1 00000	05326 LXA +0 53400 2 05577	05327 CLA +0 50000 1 03513
05330 FSB +0 30200 2 03513	05331 STD +0 60100 0 03514		
J=J+2			
05332 CLA +0 50000 0 02624	05333 ADD +0 40000 0 04341	05334 STD +0 60100 0 02624	*037
TRANSFER TO NN			
05335 TRA +0 02000 0 02631			*038
END OF CONDITIONAL			
X=XA(J+1)-XA(J-1)			
05336 CLA +0 50000 0 02624	05337 SUB +0 40200 0 04340	05340 STD +0 60100 0 05577	05341 CLA +0 50000 0 02624
05342 ADD +0 40000 0 04340	05343 PAX +0 73400 1 00000	05344 LXA +0 53400 2 05577	05345 CLA +0 50000 1 03175
05346 FSB +0 30200 2 03175	05347 STD +0 60100 0 03176		
Y=YA(J+1)-YA(J-1)			
05350 CLA +0 50000 0 02624	05351 SUB +0 40200 0 04340	05352 STD +0 60100 0 05577	05353 CLA +0 50000 0 02624
05354 ADD +0 40000 0 04340	05355 PAX +0 73400 1 00000	05356 LXA +0 53400 2 05577	05357 CLA +0 50000 1 03513
05360 FSB +0 30200 2 03513	05361 STD +0 60100 0 03514		
J=J+1			
05362 CLA +0 50000 0 02624	05363 ADD +0 40000 0 04340	05364 STD +0 60100 0 02624	*042
OTHERWISE			
05365 TRA +0 02000 0 05404			*043
X=XA(J)-XA(J-1)			
05366 CLA +0 50000 0 02624	05367 SUB +0 40200 0 04340	05370 LXA +0 53400 1 02624	05371 PAX +0 73400 2 00000
05372 CLA +0 50000 1 03175	05373 FSB +0 30200 2 03175	05374 STD +0 60100 0 03176	
Y=YA(J)-YA(J-1)			
05375 CLA +0 50000 0 02624	05376 SUB +0 40200 0 04340	05377 LXA +0 53400 1 02624	05400 PAX +0 73400 2 00000
05401 CLA +0 50000 1 03513	05402 FSB +0 30200 2 03513	05403 STD +0 60100 0 03514	
END OF CONDITIONAL			
NN			
R=SQRT.(X*X+Y*Y)			
05404 LDQ +0 56000 0 03514	05405 FMP +0 26000 0 03514	05406 STD +0 60100 0 05577	05407 LDQ +0 56000 0 03176
05410 FMP +0 26000 0 03176	05411 FAD +0 30000 0 05577	05412 STD +0 60100 0 05577	05413 ISX +0 07400 4 00010
05414 TXH +3 00000 0 05577	05415 STD +0 60100 0 02636		
THETA=270.-ATN1.(Y,X)*360./6.2831853			
05416 ISX +0 07400 4 00000	05417 TXH +3 00000 0 03514	05420 TXH +3 00000 0 03176	05421 STD +0 60100 0 05577
05422 LDQ +0 56000 0 05577	05423 FMP +0 26000 0 04347	05424 FDP +0 24100 0 04344	05425 STD +0 60000 0 05577
05426 CLA +0 50000 0 04346	05427 FSB +0 30200 0 05577	05430 STD +0 60100 0 02641	
WHENEVER THETA.L.0,THETA.360.+THETA			
05431 CLA +0 50000 0 04337	05432 ORA -0 50100 0 04350	05433 FAD +0 30000 0 04350	05434 CHS +0 76000 0 00002
05435 ADD +0 40000 0 02641	05436 TZE +0 10000 0 05443	05437 TPL +0 12000 0 05443	05440 CLA +0 50000 0 04347
05441 FAD +0 30000 0 02641	05442 STD +0 60100 0 02641		
PRINT FORMAT PP,J,R,THETA,ZA(J),ZB(J),ZC(J)-2A(J)-2B(J)-2C(J)			
IJ-1),ZA(J)-2B(J),X,Y,XA(J),YA(J)			
*050			
*050			

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05443 TSX +0 07400 4 00003 05444 STR -1 00001 0 02635 05445 STR -1 00000 0 02624 05446 STR -1 00000 0 02636
05447 STR -1 00000 0 02641 05450 CLA +0 50000 0 02624 05451 SUB +0 40200 0 05452 05452 TXH +3 00000 0 04025
05453 STA +0 62100 0 05454 05454 STR -1 00000 0 00000 05455 CLA +0 50000 0 02624 05456 SUB +0 40200 0 05457
05457 TXH +3 00000 0 04336 05460 STA +0 62100 0 05461 05461 STR -1 00000 0 00000 05462 CLA +0 50000 0 02624
05463 SUB +0 40200 0 04340 05464 LXA +0 53400 1 02624 05465 PAX +0 73400 2 00000 05466 CLA +0 50000 1 04025
05467 FSB +0 30200 2 04025 05470 STD +0 60100 0 05577 05471 STR -1 00000 0 05577 05472 CLA +0 50000 0 02624
05473 SUB +0 40200 0 04340 05474 LXA +0 53400 1 02624 05475 PAX +0 73400 2 00000 05476 CLA +0 50000 1 04336
05477 FSB +0 30200 2 04336 05500 STD +0 60100 0 05577 05501 STR -1 00000 0 05577 05502 LXA +0 53400 1 02624
05503 LXA +0 53400 2 02624 05504 CLA +0 50000 1 04025 05505 FSB +0 30200 2 04336 05506 STD +0 60100 0 05577
05507 STR -1 00000 0 05577 05510 STR -1 00000 0 03176 05511 STR -1 00000 0 03514 05512 CLA +0 50000 0 02624
05513 SUB +0 40200 0 05514 05514 TXH +3 00000 0 03175 05515 STA +0 62100 0 05516 05516 STR -1 00000 0 00000
05517 CLA +0 50000 0 02624 05520 SUB +0 40200 0 05521 05521 TXH +3 00000 0 03513 05522 STA +0 62100 0 05523
05523 STR -1 00000 0 00000 05524 STR -1 00000 0 00000
C(J)=R
05525 LXA +0 53400 1 02624 05526 CLA +0 50000 0 02636 05527 STD +0 60100 1 02612
MM CONTINUE
05530 TRA +0 02000 0 05160
PRINT FORMAT XX,AC(1)...AC(N)
05531 TSX +0 07400 4 00003 05532 STR -1 00001 0 03202 05533 CLA +0 50000 0 02632 05534 SUB +0 40200 0 05535
05535 TXH +3 00000 0 00324 05536 ALS +0 76700 0 00022 05537 STD +0 62200 0 05540 05540 STR -1 00000 0 00323
05541 STR -1 00000 0 00000
PUNCH FORMAT DATA,N,C(1)...C(N)
05542 TSX +0 07400 4 00004 05543 STR -1 00001 0 02614 05544 STR -1 00000 0 02632 05545 CLA +0 50000 0 02632
05546 SUB +0 40200 0 05547 05547 TXH +3 00000 0 02612 05550 ALS +0 76700 0 00022 05551 STD +0 62200 0 05552
05552 STR -1 00000 0 02611 05553 STR -1 00000 0 00000
WHENEVER LIC.NE.0.
05554 CLA +0 50000 0 02627 05555 SUB +0 40200 0 04337 05556 TZE +0 10000 0 05574
EXECUTE SMOTH.(C,ZB,N)
05557 TSX +0 07400 4 00007 05560 TXH +3 00000 0 02612 05561 TXH +3 00000 0 04336 05562 TXH +3 00000 0 02632
PRINT FORMAT XX,ZB(1)...ZB(N)
05563 TSX +0 07400 4 00003 05564 STR -1 00001 0 03202 05565 CLA +0 50000 0 02632 05566 SUB +0 40200 0 05567
05567 TXH +3 00000 0 04336 05570 ALS +0 76700 0 00022 05571 STD +0 62200 0 05572 05572 STR -1 00000 0 04335
05573 STR -1 00000 0 00000
END OF CONDITIONAL
TRANSFER TO START
05574 TRA +0 02000 0 02640
INTEGER N,I,J,K
VECTOR VALUES DIM=3,1,2,200
02617 +0 00000 0 00310 02620 +0 00000 0 00002 02621 +0 00000 0 00001 02622 +0 00000 0 00003
VECTOR VALUES CON=$15,2F10.0*$
02300 +0 03300 5 46060 02301 +3 10573 0 22601
VECTOR VALUES DATA=$15/(16F5.1)*$
02613 +2 60533 0 13454 02614 +3 10561 7 40106
VECTOR VALUES DATA=$16F5.1*$
02615 -1 46060 6 06060 02616 +0 10626 0 53301
VECTOR VALUES XX=$1H0/1H,14F8.1,F7.1)*$
03177 +3 30134 5 46060 03200 +1 03301 7 32607 03201 +3 06073 0 10426 03202 +0 13000 6 17401
VECTOR VALUES UU=$1H4,54,1HJ,S7,1HR,S9,1HO,S8,2HZA,S8,2HZB,S7
1,5HZA-ZA,S5,5HZA-ZB,S7,1HU,S9,1HV,S9,1HX,S9,1HY/1H
2+,S22,1H-$
02642 +0 27301 3 04054 02643 +0 13020 7 36202 02644 +1 17301 3 07061 02645 -3 30130 6 77362
02646 +0 13065 7 36211 02647 +3 06473 6 21173 02650 02650 +2 27362 0 77301 02651 +0 53071 2 14071
02652 -3 12273 6 20573 02653 -3 30530 7 12240 02654 -0 07121 7 36205 02655 +0 77305 3 07121
02656 +0 23071 2 27362 02657 -3 12173 6 21073 02660 -3 36210 7 30230 02661 -2 21173 0 13000
02662 +0 77301 3 05173 02663 -3 30130 4 17362 02664 +0 13004 7 36204
VECTOR VALUES PP=$1H,15,11F10.1)*$
02633 +3 30154 6 06060 02634 -3 30101 2 60100 02635 +0 13060 7 33105
DIMENSION A(1000,DIM),C(200),XA(200),YA(200),ZB(200),
1AC(200)
VECTOR VALUES JUMP=$1H1*$
02625 +0 13001 5 46060
END OF PROGRAM
05575 TSX +0 07400 4 00011

```

MAD (12 MAR 1962 VERSION) PROGRAM LISTING

```

EXTERNAL FUNCTION (A,B,C)
ENTRY TO SMOTH,
  R(1)=A(1), R(2)=B(1), R(3)=C(1)
  THROUGH Q FOR I=2,11, Q=C-1
  WHENEVER A(I)=1, E=0.08*A(I)-E, 0.1*TRANSFER TO Q
  R(I)=0.25*A(I)-1)+0.5*A(I)+0.25*A(I+1)
  Q CONTINUE
  R(1)=0.75*A(1)+0.25*A(2)
  R(2)=0.25*A(2)+0.75*A(3)
  FUNCTION RETURN
  INTEGER I,J,C
END OF FUNCTION

```

MAD PROGRAM TYPE 12 MAR 1962 (ALL NUMBERS ARE OCTAL)

NO. OF LOCATIONS 00204 TRA VECTOR SIZE 00001 TRA VECTOR STARTS 00000 ENTRY PT. 00013 ERASABLE STARTS 77777

PROGRAM IS AN EXTERNAL FUNCTION. THE FOLLOWING ARE ENTRIES

SMOTH 00014

VARIABLE STORAGE

I 00002

J 00003

FUNCTION DICTIONARY

ZERO 00000

ABSOLUTE CONSTANTS

00004 +000000000000

00011 +200600000000

00012 +233000000000

STATEMENT DICTIONARY

00003 TXL -300116000115

\$DATA

04/17/62 004575

MAP

```

ERROR 00000* SYSTEM 00000* SPRINT 00000* SKIP6 00000* SCALUS 00000* OPINUT 00000*
-EXIT 00000* (MAIN) 10000 SMITH 15601 -ERR 16005* -IOH 16061* -READ 20267*
-PRINT 20372* -PUNCH 20433* ZERO 20506* SORT 20542* COS 20620* SIN 20620*
ATNL 21017* -MTX 21212* (PRG) 21245 (JSHUT) 75414 (ERAS) 77767
54141 LOCKS. CAN BE SAFELY USED IN EXPANDING PROG. (OCTAL)

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